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Research Article

Elastic-Plastic Stress Analysis of Steel Fiber Reinforced Composite Plates Under Axial Load

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Abstract

Composite materials, obtained by combining two or more materials; It is defined as a new type of material with high strength, high rigidity and lightness. Composite plates are structural elements that are used in machines and structures under different loads, consist of at least two types of materials and can be produced in various constructions. In this study, elastic-plastic stress analysis of polymer matrix continuous fiber reinforced composite plate under axial load was solved with Airy Stress Function proposed as a 5th order non-uniform polynomial to solve the elasticity problem. Polyethylene matrix composite reinforced with steel fibers was taken as the plate material and the material was accepted as ideal elastic-plastic. Tsai-Hill Yield Criterion was used for the plastic solution. According to the results of the analysis, as the fiber angle increased in the composite plate, the plastic stress limit decreased, the increase in the fiber angle decreased the plastic stress limit, and the decrease in the plastic stress limit caused the residual stresses to increase.

Keywords: Composite plates, Axial load, Elastic plastic stress, Airy stress function.

1. Introduction

Composite material is a building material obtained by macroscopically combining two or more materials that do not dissolve in each other. The assembly at the macroscopic level is carried out in order to combine the best properties of the components in one material.

Reinforcement phase fibers from the elements that make up the composite material can be in the form of particles. The matrix phase is generally continuous. As a result of the production of composite material, properties such as high strength per weight, high fatigue resistance, high corrosion resistance, high wear resistance, good thermal and thermal conductivity, and aesthetics are provided in the material. For the formation of all these properties, factors such as the ideal matrix and reinforcement material pair, the production technique and their optimization are important [9].

If the deformations that occur when a load is applied to a material disappear when the load is removed, this material is called elastic material. The maximum stress value of the material is also defined as the elastic limit of the material. It is accepted that the material, which is called ideal elastic-plastic, shows full elastic before yield point and full plastic after yield point. The material used in this study is also the ideal elastic-plastic material.

Bahei-El Din & Dvorak [1] analyzed the elastic-plastic behavior of symmetric metal-matrix composite sheets for the plane stress state. Chu et al. [2] using the Airy Stress Function, stress and bending analysis were performed for the FGM beam under in-plane load. Daehn et al. [3] investigated the non-axial deformation properties of alumina and aluminum composites both experimentally and analytically using the finite element method. Daounaji et al. [4] studied stress analysis with Airy Stress Function in functionally graded cantilever beam with off-axis loading. Ding et al. [5] carried out the stress analysis of a fixed beam from both ends under a uniformly distributed load, using the Airy Stress Function and the finite element method. Fraternali & Bilotti [6] developed a finite element model for stress analysis of layered curved beams. Gahleitner & Schoeftner [7] studied the stress analysis of functionally graded beam with Airy Stress Function. Kenny & Marchetti [10] investigated the elastic-plastic behavior of thermoplastic composite sheets under periodic loading, numerically and experimentally. Meguid [12] investigated the elastic-plastic stress density of cantilever elements with different geometries under certain load. Rastgoo & Amirian [13] carried out stress analysis for different cross-section ratios of a fixed beam under parabolic loading using the Airy Stress Function. Sehlström et al. [15] carried out stress analysis of shell elements with different designs with the Airy Stress Function. Shang et al. [16] studied stress analysis with Airy Stress Function for planar orthotropic problem. Subramanian [17] performed the bending analysis of symmetrical laminated beams using the finite element method. Zhan & Liu [18] investigated the stress analysis of beams under the effect of uniformly distributed load, fixed at one end, fixed at both ends and simply supported, using Airy Stress Function and computer aided finite element method.

In this study, the elastic-plastic stress analysis of the composite plate under the axial load was performed with the Airy Stress Function, and the effect of different fiber angles on the results was examined.

2. Materials and Methods

2.1. Properties of Composite Plate

The steel fiber reinforced composite plate under axial load, for which stress analysis was performed in this study, is shown in Figure 1.

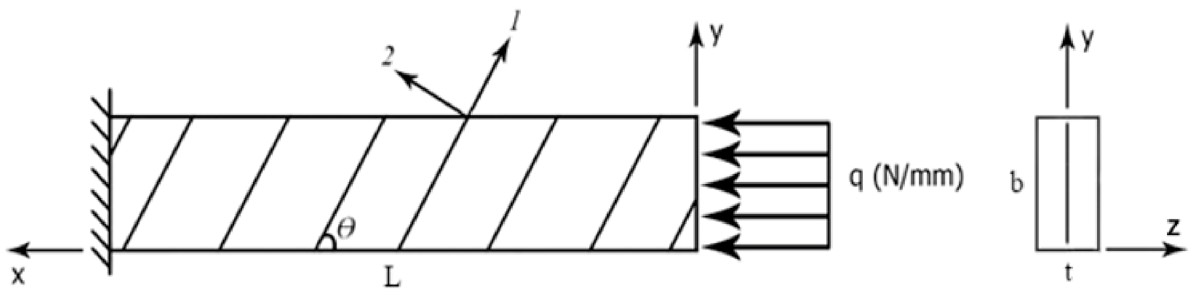

Figure 1 Plate Under Axial Load

Boundary conditions of the plate in Figure 1 [11]:

$$y = 0 \quad \sigma_y = 0 \tag{1}$$

$$y = 0 \quad \tau_{xy} = 0 \tag{2}$$

$$\int_0^y \sigma_x dA = -Q \tag{3}$$

$$\int_0^y \tau_{xy} dA = 0 \tag{4}$$

dA is the cross-sectional area of the plate:

$$dA = tdy \tag{5}$$

The dimensions of the polyethylene composite plate reinforced with steel fibers are given in table 1 and its mechanical properties are in table 2.

Table 1 Load applied to the composite plate and selected dimensional properties

q (N/mm)	L (mm)	b (mm)	t (mm)
450	300	25	10

Table 2 Mechanical properties of steel fiber composite plate [14]

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	X (MPa)	Y(MPa)	M (MPa)
85	74	30	0.3	230	24	48.9

2.2. Elastic-Plastic Stress Analysis

The purpose of solving elasticity problems with the Airy Stress Function is to find a stress function that satisfies both the equilibrium equations and the conformity equations. The biharmonic equation that satisfies the equilibrium equations and conformity conditions is as follows [11]:

$$\bar{S}_{22}\frac{\partial^4\phi}{\partial x^4} - 2\bar{S}_{26}\frac{\partial^4\phi}{\partial x^3\partial y} + (2\bar{S}_{12} + \bar{S}_{66})\frac{\partial^4\phi}{\partial x^2\partial y^2} - 2\bar{S}_{16}\frac{\partial^4\phi}{\partial x\partial y^3} + \bar{S}_{11}\frac{\partial^4\phi}{\partial y^4} = 0 \tag{6}$$

Where, the stress function. The equilibrium equation given by Jones [8] is:

$$\begin{bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66}\end{bmatrix}\begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix} \tag{7}$$

Where, the elements of the transformed reduced elasticity matrix:

$$\bar{S}_{11} = S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4 \quad (8)$$

$$\bar{S}_{12} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2 \quad (9)$$

$$\bar{S}_{22} = S_{11}s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}c^4 \quad (10)$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c \quad (11)$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3 \quad (12)$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4) \quad (13)$$

Where, $c = \cos(\theta)$ ve $s = \sin(\theta)$

$$S_{11} = \frac{1}{E_1} \quad S_{12} = -\frac{\upsilon_{12}}{E_1} \quad S_{22} = \frac{1}{E_2} \quad S_{66} = \frac{1}{G_{12}} \quad (14)$$

In this study, the function determined to satisfy both the boundary conditions of the plate and the equilibrium equations is:

$$\phi = \frac{x^2y^3}{6}d + \frac{y^5}{20}f + \frac{xy^4}{12}e + \frac{x^2y}{2}b + \frac{xy^2}{2}c \quad (15)$$

The stress components are:

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = dx^2y + ndy^3 + mdxy^2 + cx \quad (16)$$

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = d\frac{y^3}{3} + by \quad (17)$$

$$\tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = -dxy^2 - md\frac{y^3}{3} - bx - yc \quad (18)$$

When equation 15 is applied in equation 6:

$$e = \frac{2\bar{S}_{16}}{\bar{S}_{11}} d \tag{19}$$

Where, $\frac{2\bar{S}_{16}}{\bar{S}_{11}} = m$

$$e = md \tag{20}$$

$$f = \frac{2\bar{S}_{16}m - 2\bar{S}_{12} - S_{66}}{3\bar{S}_{11}} d \tag{21}$$

Where $\frac{2\bar{S}_{16}m - 2\bar{S}_{12} - S_{66}}{3\bar{S}_{11}} = n$

$$f = nd \tag{22}$$

Unknown constants when equations 1-5, equations 16-18 are applied:

$$c = \frac{-4dxy - dy^2m + 2xy}{6} \tag{23}$$

$$d = \frac{-36Q}{12x^2y^2t + 9y^4nt + 10xy^3mt} \tag{24}$$

$$e = \frac{-36Qm}{12x^2y^2t + 9y^4nt + 10xy^3mt} \tag{25}$$

$$f = \frac{-36Qn}{12x^2y^2t + 9y^4nt + 10xy^3mt} \tag{26}$$

In the solution, the Tsai-Hill criterion was taken as the yield criterion for the composite material. According to the Tsai-Hill criterion, the equivalent stress in the parent material direction is:

$$\sigma^{-2} = \sigma_1^2 - \sigma_1\sigma_2 + \frac{X^2}{Y^2}\sigma_2^2 + \frac{X^2}{S^2}\tau_{12}^2 = X^2 \tag{27}$$

Where, X is the yield point in the 1 material direction; Y is yield point in 2 material directions; The shear in the S 1-2 plane is the yield strength. The stress components in the material directions are written as:

$$\sigma_1 = \sigma_x \cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta \tag{28}$$

$$\sigma_2 = \sigma_x \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta \tag{29}$$

$$\tau_{12} = -\sigma_x \sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \tag{30}$$

When the stress components and plate boundary conditions in the principal material directions are applied to the Tsai-Hill theory, the equation that gives the yield tensile strength, that is, the plastic stress limit, is obtained as follows [14]:

$$\sigma_{x(p)} = \frac{X}{\sqrt{\cos^4(\theta) - \sin^2(\theta)\cos^2(\theta) + ((X^2\sin^4(\theta))/Y^2) + ((X^2\sin^2(\theta)\cos^2(\theta))/M^2)}} \tag{31}$$

If the stresses occurring in the material under the influence of a certain load are less than the yield tensile strength, the stresses in the material are also removed after the load is removed, so this stress is called elastic stress. If the stresses formed in the material exceed the yield tensile strength, plastic stresses occur and residual stresses occur in the material after the load is removed.

3. Results

The analysis results in table 3 were found by applying the conditions in table 1 to the formulation obtained by applying equations 23-26 to equation 15. Table 3 shows the normal stresses, plastic stress limits and residual stresses at the top surface of the plate corresponding to the fiber angles of 0, 30, 45, 60 and 90 degrees.

Table 3 Stress analysis results

θ (degree)	x (mm)	σ_x (MPa)	$\sigma_{x(p)}$ (MPa)	$\sigma_{x(r)}$ (MPa)
0^0	50	-37.699	-230.000	-
	100	-43.373	-230.000	-
	150	-44.291	-230.000	-
	200	-44.604	-230.000	-
	250	-44.747	-230.000	-
	300	-44.825	-230.000	-
30^0	50	-35.957	-71.839	-
	100	-42.658	-71.839	-
	150	-43.821	-71.839	-
	200	-44.250	-71.839	-
	250	-44.463	-71.839	-
	300	-44.587	-71.839	
45^0	50	-36.647	-41.117	-
	100	-42.842	-41.117	-1.725
	150	-43.921	-41.117	-2.804
	200	-44.319	-41.117	-3.202
	250	-44.515	-41.117	-3.398
	300	-44.628	-41.117	-3.511
60^0	50	-37.609	-30.838	-6.771
	100	-43.161	-30.838	-12.323
	150	-44.114	-30.838	-13.276
	200	-44.457	-30.838	-13.619
	250	-44.623	-30.838	-13.785
	300	-44.718	-30.838	-13.880

90^0	50	-38.774	-24.000	-14.774
	100	-43.589	-24.000	-19.589
	150	-44.383	-24.000	-20.383
	200	-44.655	-24.000	-20.655
	250	-44.780	-24.000	-20.780
	300	-44.847	-24.000	-20.847

4. Discussion and Conclusion

In this study, elastic-plastic stress analysis was carried out according to different fiber angles with Airy Stress Function of polymer matrix composite plate under axial load. The stresses on the uppermost surface of the plate were found.

According to the analysis results;

• As the fiber angle increased in the composite plate, the plastic stress limit decreased

• As it approaches the fixed end from the free end of the composite plate, the normal stresses and, accordingly, the residual stresses increased

• When the studies in the literature for the plate under the effect of bending were examined, it was found that the stresses in the plate decreased as the fiber angle increased

• Increasing the plate fiber angle under the axial load increases the normal stresses, albeit at a small value

• Increasing the plate fiber angle under the axial load reduces the plastic stress limit

• The decrease in the plastic stress limit is a negative situation that causes the residual stresses to increase

• It is appropriate to have a fiber angle when making fiber arrays on the plate that exerts axial load alone. Considering all these data, the appropriate plate construction should be selected.

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